Quantized 2d Dilaton Gravity and Black Holes

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abstract

We have examined a modified dilaton gravity whose action is separable into the kinetic and the cosmological terms for the sake of the quantization. The black hole solutions survive even in the quantized theory, but the ADM mass of the static solution is unbounded from below. Quantum gravitational effect on the interacting matter field is also examined.

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1 Introduction

The quantum theories of the dilaton gravity (DG) have been studied by many authors [1][2][3][4] in order to resolve the problems related to the Hawking radiation of black holes [5]. The necessity of the full quantization of DG has been pointed out because of a singularity found in a semiclassical approach based on the $1/N$ expansion [6]. The quantized model, which has previously proposed [1][2][3], has been obtained according to the principle of conformal invariance of the quantized theory. The resultant effective action is written in the form of a non-linear σ-model where the cosmological term is added as a marginal operator to a simple conformally invariant theory. The field variables in this effective action are constructed so that they reproduce a semi-classically quantized dilaton gravity, in which the necessary Liouville terms are included, at the weak coupling limit, $\mathrm{e}^{\phi} << 1$. However the original DG is an interacting system of the dilaton, so the path integral measure for the dilaton field would not be the usual anomaly term which is given for the conformal fields without interactions.

Here we take care of this point for the quantized measure, and a new quantization scheme is proposed by giving a modified dilaton gravity which is equivalent to the original DG classically. The action of our model can be separated to two parts, the kinetic and the cosmological terms. The kinetic part is easily quantized according to the method of [7][8]. The cosmological term can be regarded as a perturbation for small cosmological constant (μ). Then we can use the technique used in [9] for the quantization of this theory. Although our method is valid for small μ, this is not an obstacle to study the problems related to the black holes since the qualitative properties of DG does not depend on the magnitude of μ as far as it is non-zero. Here we give the quantized DG up to the $O(\mu^2)$, and the qualitative properties of the solutions of the equations of motion are discussed. Another purpose in this paper is to examine the quantum effects of DG on the interacting matter fields, especially on the renormalization group equations.

Usually N conformal matter fields are added to DG. But they are not suitable for seeing the gravitational effects on themselves. Previously we have shown an universal gravitational effect on the renormalization group equations of the interacting matter fields [9]. Then we consider here a matter field which has its self-interaction in order to study the same quantity. It can be shown that the same form of the corrections are found also in the case of DG. But the physical scale factor, which appears in the formula, is different from the previous case. The reason is that this scale factor is depending on the gravitational models. This difference is also seen in the string susceptibility (γ_s) of the theory. We can see that γ_s in DG for small μ is real for any

number of conformal matter fields. This is also pointed out in [4] by a semi-classical approach.

2 A model equivalent to dilaton gravity

Our starting point is the following DG [5] which is written by the metric, dilaton ϕ and N conformal matter fields f_i ($i = 1 \sim N$);

$$S_{\text{dil}} = \frac{1}{4\pi} \int d^2 z \sqrt{g} e^{-2\phi} \left\{ [R + 4(\nabla\phi)^2 + 4\mu^2] - \frac{1}{2} \sum_{i=1}^{N} (\nabla f_i)^2 \right\}, \tag{1}$$

where ∇_μ denotes the covariant derivative with respect to the 2d metric $g_{\mu\nu}$. As long as the cosmological constant μ is nonzero, this action provides non-trivial solutions, (i) static black hole solutions and (ii) the one which describes the formulation of a black hole by the infall of an arbitrary pulse of massless scalar matter.

This has been extensively explored as a toy model of four dimensional gravity in order to make clear the problems concerned with the black holes. Since the properties of DG mentioned above do not depend on the the magnitude of the parameter μ^2, we consider the case of very small μ^2 in order to use it as a perturbation parameter. S_{dil} has still a complicated form for ϕ, so we consider a slightly modified form, which is classically equivalent to (1) and separable to kinetic and cosmological terms. It is written by introducing an auxiliary field $\lambda(z)$,

$$Z = \int D\lambda \tilde{Z}(\lambda), \tag{2}$$

$$\tilde{Z}(\lambda) = \int Dg_{\mu\nu} D\omega D f_i D\chi e^{-S}, \tag{3}$$

where the action is written by the kinetic part S_0 and the 'cosmological' term S_{cos}, $S = S_0 + S_{\text{cos}}$ and

$$S_0 = -\frac{1}{4\pi} \int d^2 z \sqrt{g} [\chi R + 4(\nabla\omega)^2 - \frac{1}{2} \sum_{i=1}^{N} (\nabla f_i)^2], \tag{4}$$

$$S_{\text{cos}} = -\frac{1}{4\pi} \mu^2 \int d^2 z \sqrt{g} [(\lambda + 4)\omega^2 - \lambda\chi]. \tag{5}$$

We can see the equivalency of this model and (1) at the classical level by integrating over λ and χ in (2) and by interpreting ω as $e^{-\phi}$. So it is trivial to see that the classical properties of this model are the same with that of the original DG (1). Although it is not a simple problem to quantize DG in the form (1), we can proceed a systematic procedure of quantization in our equivalent model, as shown below.

It can be seen that S_0 is written by the kinetic terms only in the conformal gauge and its fully quantized form can be obtained easily as shown below. It should be noticed that $(S =)S_0$ is essentially equivalent to the model of constant curvature, $R = 0$ in this case, which is proposed [10] as a non-critical string model for arbitrary dimensions. However our model is different from the constant curvature model because of the existence of $S_{\cos}$. The procedure of the quantization for $\mu^2 \neq 0$ can be performed by treating $S_{\cos}$ as a small perturbation according to the method of [9].

Our strategy is as follows. First, obtain the effective action in the form,

$$\tilde{Z}(\lambda) = \mathrm{e}^{-S_{\mathrm{eff}}}, \tag{6}$$

then examine the properties of the quantized theory by integrating out λ in (2) in the final step. In $\tilde{Z}(\lambda)$, λ is treated as an external field. Here we take the conformal gauge, $g_{\mu\nu} = \mathrm{e}^{2\rho}\hat{g}_{\mu\nu}$ with a fiducial metric $\hat{g}_{\mu\nu}$. Due to the metric invariance, it is well known that to obtain the fully quantized action is equivalent to getting the one which is conformally invariant with respect to the fiducial metric [8]. According to this idea, the effective action is obtained for $\mu^2 = 0$ by adopting the following definition of the norms for each variables, $X^a = (\rho, \chi, \omega, f_i)$ where $a = 0, 1, \cdots, 2 + N$,

$$\|\delta X^a\|^2 = \int d^2 z \sqrt{\hat{g}}(\delta X^a)^2. \tag{7}$$

By dividing $\tilde{Z}$ by the gauge volume, the result is given by the following non-linear σ-model form,

$$\tilde{Z}_q|_{\mu^2=0} = \int D_{\hat{g}}\rho D_{\hat{g}}\omega D_{\hat{g}}f_i D_{\hat{g}}\chi \mathrm{e}^{-S_{\mathrm{eff}}^{(0)}}, \tag{8}$$

$$S_{\mathrm{eff}}^{(0)} = \frac{1}{4\pi} \int d^2 z \sqrt{\hat{g}} \left[\frac{1}{2} G_{ab}^{(0)}(X)\hat{g}^{\alpha\beta}\partial_\alpha X^a \partial_\beta X^b + \hat{R}\Phi^{(0)}(X) + T^{(0)}(X) \right], \tag{9}$$

and

$$G_{ab}^{(0)} = \begin{pmatrix} \kappa & -2 & & \\ -2 & 0 & & \\ & & -8 & \\ & & & \mathbf{1} \end{pmatrix}, \quad \Phi^{(0)} = \frac{1}{2}\kappa\rho - \chi, \quad T^{(0)} = 0, \tag{10}$$

where $\mathbf{1}$ in $G_{ab}^{(0)}$ denotes the unit matrix of dimension N and

$$\kappa = \frac{23 - N}{3}. \tag{11}$$

As for the effective action for $\mu^2 \neq 0$, it can be obtained perturbatively by expanding S_{eff} in the power series of μ^2 as follows,

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$$S_{\text{eff}} = S_{\text{eff}}^{(0)} + \mu^2 S_{\text{eff}}^{(1)} + \mu^4 S_{\text{eff}}^{(2)} + \cdots \tag{12}$$

$$= \frac{1}{4\pi} \int d^2 z \sqrt{\hat{g}} \left[\frac{1}{2} G_{ab}(X, \lambda) \hat{g}^{\alpha\beta} \partial_\alpha X^a \partial_\beta X^b + \hat{R} \Phi(X, \lambda) + T(X, \lambda) \right]. \tag{13}$$

where

$$G_{ab} = G_{ab}^{(0)} + \mu^4 G_{ab}^{(2)} + \cdots, \quad \Phi = \Phi^{(0)} + \mu^4 \Phi^{(2)} + \cdots,$$

$$T = T^{(0)} + \mu^2 T^{(1)} + \cdots. \tag{14}$$

It can be seen from the construction that $S_{\text{eff}}^{(1)}$ is the so-called dressed term of S_{cos} and the terms of order $0(\mu^2)$ do not appear in G_{ab} and Φ as seen below. The higher order terms are obtained by solving the equations of motion derived from the following target space action,

$$S_t(\lambda) = \frac{1}{4\pi} \int d^d X \sqrt{G} e^{-2\Phi} [R - 4(\nabla\Phi)^2 + \frac{1}{16}(\nabla T)^2 + \frac{1}{16} v(T) - \kappa], \tag{15}$$

where $d = 3 + N$ and the higher derivative terms are suppressed, and

$$v(T) = -2T^2 + \frac{1}{6}T^3 + \cdots . \tag{16}$$

∇_μ denotes the covariant derivative with respect to the metric G_{ab}. We notice here that λ is not a coordinate but only a parameter. And the equations to be solved are obtained as follows,

$$\nabla^2 T - 2\nabla\Phi\nabla T = \frac{1}{2} v'(T), \tag{17}$$

$$\nabla^2 \Phi - 2(\nabla\Phi)^2 = -\frac{\kappa}{2} + \frac{1}{32} v(T), \tag{18}$$

$$R_{ab} - \frac{1}{2} G_{ab} R = -2\nabla_a \nabla_b \Phi + G_{ab} \nabla^2 \Phi + \frac{1}{16} \nabla_a T \nabla_b T - \frac{1}{32} G_{ab} (\nabla T)^2. \tag{19}$$

where $v' = dv/dT$.

3   Quantum corrections

We firstly determine $S_{\text{eff}}^{(1)}$ or $T^{(1)}$ as the dressed operator of S_{cos} by using $S_{\text{eff}}^{(0)}$ according the idea of [11]. Two terms are contained in S_{cos}, the terms proportional to ω^2 and χ, and their dressed forms are given separately. The method is as follows.

Except for the prefactor $\mu^2(\lambda + 4)/4\pi$, the dressed term of $e^{2\rho}\omega^n$ is given as the coefficient of $\beta^n/n!$ of the marginal operator of

$$\hat{T} = e^{\alpha\rho+\beta\omega}$$

by expanding it in terms of β. The parameter α can be obtained by solving the lowest order equation of (17) with respect to μ^2, and it is written as,

$$G^{(0)\,ab}[\partial_a\partial_b - 2\partial_a\Phi^{(0)}\partial_b + 2]\hat{T} = 0. \tag{20}$$

This equation gives,

$$\alpha = -\frac{1}{8}\beta^2 + 2, \tag{21}$$

Then the dressed form of $e^{2\rho}\omega^2$ is obtained as,

$$e^{2\rho}(\omega^2 - \frac{1}{4}\rho). \tag{22}$$

Similarly we can get the dressed one for $e^{2\rho}\chi$ by solving (20), where $\hat{T}$ is replaced by $\hat{T} = e^{\alpha\rho+\beta\chi}$. The result is as follows,

$$e^{2\rho}(\chi - 2\rho). \tag{23}$$

As a result, we obtain

$$S_{\text{eff}}^{(1)} = -\frac{1}{4\pi}\int d^2z\sqrt{\hat{g}}e^{2\rho}[(\lambda+4)(\omega^2 - \frac{1}{4}\rho) - \lambda(\chi - 2\rho)]. \tag{24}$$

The next task is to solve eqs.(18) and (19). It should be noted that the lowest order ($O(1)$) equation of (18),

$$G^{(0)\,ab}(\partial_a\partial_b\Phi^{(0)} - 2\partial_aT^{(0)}\partial_bT^{(0)}) = -\frac{\kappa}{2}, \tag{25}$$

provides $\Phi^{(0)}$ which is given in (10). The next order begins from $O(\mu^4)$ which is needed to balance with the substituted $T^{(1)}$ in eqs.(18) and (19). Then the equations for $G_{ab}^{(2)}$ and $\Phi^{(2)}$ can be written down, but they include too many terms. So we can solve them by restricting the space of variables to that of ρ, χ and ω since f_i decouple from them. And we finally obtain seven equations by which we could solve seven unknown functions $G_{ab}^{(2)}$ ($a, b = 0, 1, 2$) and $\Phi^{(2)}$. But the equations contain second derivative terms, so we need many boundary conditions to obtain a meaningful solution from them. From those solutions, various counter terms needed for the theory can be seen and they are necessary to solve the quantum mechanical black hole solutions up to the order $O(\mu^4)$. However our purpose is to examine the

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properties of the theory up to $O(\mu^2)$, so we do not need their explicit form which will be given elsewhere.

Instead of giving the higher order counter terms of μ^2, we examine here the quantum effects of DG on the matter field by adding an interaction of a matter field (with a coupling constant γ). We restrict our attention on this gravitational effect on the renormalization group property of the matter fields. It is seen as follows by solving (18) and (19) up to γ^2 by assuming $\mu^2 << \gamma$.

Define f_i as $\{f_j \ (j = 1 \sim N - 1), \ \varphi = f_N\}$, and add the following interaction term of φ,

$$S_{\text{int}} = \frac{1}{4\pi}\gamma \int \sqrt{g}V(\varphi) \tag{26}$$

to the original action S. Here we consider the Sine-Gordon model as an example,

$$V(\varphi) = \cos(p\varphi), \tag{27}$$

which has been previously examined in the usual 2d quantum gravity without dilaton. Then we can see the difference between DG and the usual 2d gravity by comparing our results obtained here and the one given in the previous work [9].

In order to simplify the problem, the fields X^a in (10) are changed as follows,

$$\rho = \frac{1}{\sqrt{\kappa}}(\rho' + \sqrt{2}\kappa'), \quad \chi = \sqrt{\frac{\kappa}{2}}\chi', \quad \omega = \frac{1}{2\sqrt{2}}\omega'. \tag{28}$$

Then the form of $S_{\text{eff}}^{(0)}$, which can be obtained in the limit of $\gamma = \mu^2 = 0$, can be written in the same form with (9) but with the different $X'^a = (\rho', \chi', \omega', f_j, \varphi)$, $G_{ab}^{(0)}$ and $\Phi^{(0)}$ which are given as,

$$G_{ab}^{(0)} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & \mathbf{1} \end{pmatrix}, \quad \Phi^{(0)} = \sqrt{\frac{\kappa}{2}}\rho'. \ T^{(0)} = 0, \tag{29}$$

In the following we suppress the prime of X^a for the sake of the brevity.

The calculations are performed according to the procedure given above. But we are considering here the perturbation in γ instead of μ^2 under the assumption, $1 >> \gamma >> \mu^2$. Firstly, we obtain the dressed operator $\hat{V}$ by solving the lowest equation of (17). It should be solved by taking the following form

$$\hat{V}(\varphi) = V(\varphi)e^{\delta(\rho + \sqrt{2}\chi)} = \cos(p\varphi)e^{\delta(\rho + \sqrt{2}\chi)}. \tag{30}$$

Here we should notice that the exponent of the dressed factor is changed due to the shift, which is given in (28), of the original variable ρ. Equation (17) provides the

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following result,

$$\delta = \frac{2 - p^2}{\sqrt{\kappa}}. \tag{31}$$

From this we find a possible solution,

$$p = \sqrt{2}, \quad \delta = 0, \tag{32}$$

which is corresponding to the Kosterlitz-Thouless fixed point.

Nextly, we solve the equations of $O(\gamma^2)$ of (18) and (19) by substituting the solution (30) and (31). They are solved under the ansatz; (i) The functional coordinate space $\{X^a\}$ is restricted to $\{\rho, \chi, \varphi\}$ since $\hat{V}$ is in this space. (ii) Since the properties of the renormalization group equations are determined by the ρ-dependence of G_{ab} of matter fields [9], we solve the equations by taking the ansatz; $G^{(2)}_{\varphi\varphi} = h(\rho)$ and other $G^{(2)}_{ab} = 0$. Then we obtain the next seven equations,

$$(\partial^2 - 2\sqrt{\kappa}\partial_\rho)\Phi^{(2)} - \frac{\sqrt{\kappa}}{4}\partial_\rho h = -\frac{1}{16}\hat{V}^2, \tag{33}$$

$$\partial_a\partial_b\Phi^{(2)} + \frac{1}{4}[\delta^0_a\delta^0_b\partial^2_\rho + \delta^1_a\delta^1_b(\partial^2_\rho + \sqrt{\kappa}\partial_\rho)]h = \frac{1}{32}\partial_a\hat{V}\partial_{\hat{V}}, \tag{34}$$

where $\partial^2 = \partial^2_\rho - \partial^2_\chi + \partial^2_\varphi = \partial^2 = \partial^2_0 - \partial^2_1 + \partial^2_{3+N}$. They are solved as follows,

$$\Phi^{(2)} = \frac{1}{128}\hat{V}^2, \quad h = \frac{p^2}{16\sqrt{\kappa}}\rho. \tag{35}$$

This solution for $h(\rho)$ is same with the one given in [9], then the same form of the renormalization group equations of δ and p are obtained by considering them near the Kosterlitz-Thouless fixed point, $\gamma = 0, p = \sqrt{2}$. In fact, by parametrizing those parameters as, $p = \sqrt{2} + \epsilon$, the following β-functions are obtained,

$$\beta_\epsilon = \frac{\sqrt{2}}{16}\frac{2}{\eta Q}\gamma^2, \quad \beta_\gamma = 2\sqrt{2}\frac{2}{\eta Q}\epsilon\gamma,, \tag{36}$$

for the shift of the scale, $\rho \to \rho + 2dl/\eta$, where $\eta = 2/Q$ [2] is defined by the dressed factor of the cosmological constant, $e^{\eta\rho}$. The common factor $\frac{2}{Q\eta}$ in (36) represent the quantum gravitational effect on the matter β-functions. The difference from the results obtained in the previous work is the factor η, which characterizes the 2d gravitational theory.

Finally, we comment on the string susceptibility (γ_s) of DG for small μ^2. Neglecting the terms $O(\mu^2)$, we obtain γ_s according to [8],

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[2]This is obtained by solving eq.(20) for $\hat{T} = e^{\eta\rho}$ in terms of (29).

$$\gamma_s = 2 - \frac{\kappa}{2\eta}\tilde{\chi}, \quad \tilde{\chi} = \frac{1}{4\pi}\int d^2z\sqrt{\hat{g}}\hat{R}, \tag{37}$$

where $\tilde{\chi}$ is the Euler character. Here we must be careful with η which is defined through the dressed factor $e^{\eta\rho}$ in terms of the original variable ρ (not ρ'). Then we must take $\eta = 2$, which is obtained from eqs.(20) and (10) with $\hat{T} = e^{\eta\rho}$, in (37), and we obtain

$$\gamma_s = 2 - \frac{23 - N}{12}\tilde{\chi}.$$

This means that γ_s is always real for any N. This is because of the strong constraint on the curvature R. In fact, $R = 0$ for $\mu^2 = 0$. But higher order corrections of μ^2 would modify the above γ_s, and some constraint on N may appear. On this point, we will discuss elsewhere.

4   Black holes and static solution

Now we turn to the quantized dilaton gravity which includes $O(\mu^2)$ correction,

$$S_{\text{eff}} = S_{\text{eff}}^{(0)} + \mu^2 S_{\text{eff}}^{(1)}.$$

And the matter interaction term is suppressed here, e.g. $\gamma = 0$. After integrating over χ and λ, we obtain the next equations of motion for ρ and ω,

$$\partial_+\partial_-\omega^2 - \frac{\kappa - 7}{2}\partial_+\partial_-\rho = -\mu^2 e^{2\rho}(\omega^2 - \frac{2\rho + 1}{8}), \tag{38}$$

$$\frac{1}{\omega}\partial_+\partial_-\omega + \frac{1}{2}\partial_+\partial_-\rho = -\frac{1}{4}\mu^2 e^{2\rho}, \tag{39}$$

where we have introduced the light cone variables, $z^{\pm} = z^0 \pm z^1$. It is worthwhile to find an exact solution of the above equation, but the above equations are approximate since the higher order terms of $O(\mu^4)$ are suppressed. Then it is enough to find a solution accurate up to $O(\mu^2)$ for our purpose of examining (i) the possibility of the black hole solutions and (ii) the stability of static solutions.

Firstly, we study the possible black hole solution according to the following ansatz,

$$e^{-2\rho} = M - \mu^2 f(z), \quad \omega^2 = m - \mu^2 g^{(1)}(z) + \mu^4 g^{(2)}(z) + O(\mu^6), \tag{40}$$

where M, m are constants. The classical equations of motion derived from (4) and (5) give the following black hole solution in the conformal gauge,

$$e^{-2\rho} = \omega^2 = \frac{M_{\text{bh}}}{\mu^2} - \mu^2 z^+ z^-, \tag{41}$$

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where M_{bh} denotes the black hole mass, which is an arbitrary constant. We had taken our ansatz so that the form of the classical solution for ρ, which provides the black hole configuration, is preserved if we obtain $f \propto z^+ z^-$ and the higher order corrections affect only on the configuration of ω (the dilaton).

After a calculation, we can get the following solution of eqs.(38) and (39),

$$f(z) = f_0 + f_1 z^+ z^-, \quad g^{(1)}(z) = g_1 + \frac{m(1 + f_1)}{2M} z^+ z^-, \tag{42}$$

$$g^{(2)}(z) = A z^{+2} z^{-2} + B z^+ z^- + C, \tag{43}$$

where f_0, g_1 and C are arbitrary constants. A and B are the calculable constants depending on f_0 and g_1, and

$$f_1 = \frac{2}{m + \kappa - 7}(m + \frac{1}{4} \ln \frac{M}{e}). \tag{44}$$

Since the scalar curvature is represented as $R = 8\mathrm{e}^{-2\rho}\partial_+\partial_-\rho$, the above black hole solution represents the same z-dependence with the classical one except for the normalization. This means that we can find a black hole solution even in fully quantized DG. It would be an interesting problem to consider the relation between the result obtained here and an exact 2d black hole solution which was indicated by WZW models [12].

Nextly, we turn to the problem of the the stability of the static solution on which we should consider various problems. To see the stability, we study the ADM mass for the state which approaches to the linear dilaton vacuum, $\rho = 0$, $\omega = \mathrm{e}^{\mu\sigma}$, at the spacial infinity, $\sigma = (z^+ - z^-)/2 \to \infty$. Such a configuration would be written as follows,

$$\rho = \delta\rho, \quad -\ln \omega = -\mu\sigma + \delta\phi, \tag{45}$$

at large σ, where $\delta\rho$ and $\delta\phi$ decrease rapidly with σ. By substituting (45) into (38) and (39), we obtain the following linearized equations for $\delta\rho$ and $\delta\phi$,

$$\delta\rho + \frac{1}{\mu}(\partial + \frac{1}{4\mu}\partial^2)\delta\phi = \frac{1}{16}\mathrm{e}^{-2\mu\sigma}, \tag{46}$$

$$(\partial^2 + 4\mu^2)\delta\rho - 2(\partial^2 + 2\mu\partial)\delta\phi = 0, \tag{47}$$

where ∂ denotes the derivative with respect to σ. They are solved as follows,

$$\delta\rho = (\frac{1}{16} + a + b\sigma)\mathrm{e}^{-2\mu\sigma}, \quad \delta\phi = (a + b\sigma)\mathrm{e}^{-2\mu\sigma}, \tag{48}$$

where a, b are arbitrary constants. From this asymptotic solution, the ADM mass are obtained as,

$$M_{\mathrm{ADM}} = 2\mathrm{e}^{2\mu\sigma}(\mu\delta\rho + \partial_+\delta\phi - \partial_-\delta\phi) \tag{49}$$

$$= 2[b + \mu(\frac{1}{16} - a) - \mu b\sigma]. \tag{50}$$

Even if we take $b = 0$ so that the ADM mass does not diverge at $\sigma \to \infty$, M_{ADM} is unbounded from below because a is arbitrary. This disease is common to other quantized dilaton theory, and it is fatal to study the end of the black hole by considering a falling matter.

5   Conclusion

New quantization scheme has been proposed for the dilaton gravity by reformulating the original DG so that the action can be separated to the kinetic and a perturbation. The quantized theory has the black hole solution which is qualitatively same with the classical one. But the ADM mass of the static solution, which approaches to the linear dilaton vacuum at spacial infinity, is unbounded from below. This might be a sign that some nonperturbative quantum effect [13] would be necessary to approach the problem of the Hawking radiation and the end of the black holes.

The string susceptibility of DG for small μ^2 is shown to be real for any number of the conformal matters. Although the consideration of higher order corrections are necessary, it would be worthwhile to consider this DG as a non-critical string model for higher dimensions than two. We also find a quantum effect of DG on the renormalization group equations of the interacting matter fields. The results are consistent with the previous results obtained in the gravitation without dilaton field.

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